The introduction of a logical concept of meaning

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May 1997

Abstract

The traditional semantic is based on the supposition of two ontological levels — a *world* and a *language* — and understands semantic symptoms as mutual mappings between the phenomena of the world and the expressions of the language: expressions express phenomena on the one hand, phenomena are the meanings of the expressions on the other.

The assumed semantic here is fundamentally different. It presupposes that world and language cannot be separated in semiotic terms. There is only one category, namely a system of signs (called *assertions*). Semantic relations do only exist as relations between the signs of a given system. The *fundamental principle* is thus: the *meaning* of an assertion is also an assertion, and of the same system. Semantic relations do not exist as identities, established between world and language, but exclusively as differences within a net of signs. The *opposition principle* demands even more: the constituents (called *bit variables*) of the meaning of a given assertion are exactly the constituents of the sign system, which are not the constituents of the assertion itself.

But with that a formal concept of meaning is still underdetermined and further postulates are proposed, especially the *verification principle*: the meaning of a given assertion is a (most general) case, for which the assertion becomes true. In that way a *meaning function* can actually be defined. This even possesses the powerful property, that the traditional truth concept of formal logic can be completely embedded into the new developed concept of meaning, so that truth now only appears as a borderline case of meaning.

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Chapter 1

Basic concepts

1.1 Bit variables and bit values

The most elementary units of our logic are the so–called binary variables, or in short: the BIT VARIABLES[1]. A bit variable enables us to make a yes/no decision. That's all.

We express that by using the following denotations:

— A bit variable A can have one of two values: either 1 or 0.
— $A = 1$ means something like "A holds" or "A is the case". Most of the time we write again A instead of $A = 1$.
— $A = 0$ can be paraphrased by "A does not hold" or "A is not the case". This is mostly written as $\overline{A}$ or $\neg A$.
— 0 or 1 are the so–called BIT VALUES.

In a standard example that will be discussed throughout, the four bit variables H, R, S, W occur. To make it more illustrative they shall be associated with the following intuitions:

— H stands for "it is hot" of "the weather is warm".
— R stands for "it is raining".
— S stands for "it is snowing".
— W stands for "it is wet" or "wet (bad, moist) weather".

Accordingly

— $\overline{H}$ or $\neg H$ stands for "it is not hot" or "the weather is not warm".
— $\overline{R}$ or $\neg R$ stands for "it is not raining".
— $\overline{S}$ or $\neg S$ stands for "it is not snowing".
— $\overline{W}$ or $\neg W$ stands for "it (the weather) is not wet".

A set of bit variables constitutes an (assertoric logical) world. They are the ATOMS of this world. The more atoms such a world has, the more potentially manifold it is.

[1] Usually bit variables are called SENTENTIAL, PROPOSITIONAL or ASSERTION VARIABLES. But we will avoid this terminology for reasons which will become evident later on.

But the denotation atom must not be interpreted all too physically, as it stands for potential decisions, not for material units. Wittgenstein describes that in the *Tractatus logico-philosophicus* by saying in statement 1.1:

> *Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge. (The world is the total of the facts, not of the things.)*[2]

1.2 Valuations

A world consisting of n bit variables has 2^n different states, depending on the bit value each of the bit variables takes.

States of a world, given by the example set $\{H, R, S, W\}$ are for instance:

— $\overline{HR}SW$ for "it is not hot, does not rain, it is snowing, and it is wet" or

— $HR\overline{S}W$ for "it is hot, it is raining, it is not snowing, and it is wet".

Alltogether the bit variable set $\{H, R, S, W\}$ generates a world of $2^4 = 16$ different states, and these are:

$$\begin{array}{cccc}
\overline{HRSW} & H\overline{RSW} & \overline{H}R\overline{SW} & HR\overline{SW} \\
\overline{HR}S\overline{W} & H\overline{R}S\overline{W} & \overline{H}RS\overline{W} & HRS\overline{W} \\
\overline{HRS}W & H\overline{RS}W & \overline{H}R\overline{S}W & HR\overline{S}W \\
\overline{HR}SW & H\overline{R}SW & \overline{H}RSW & HRSW
\end{array}$$

But instead of states we rather talk about VALUATIONS of the set of bit variables, for a state is nothing else than an assignment each of a bit value to each of the given bit variables.

1.3 Theories

To describe a world completely, the specification of the atoms and states is insufficient. It is also necessary to distinguish the possible and the impossible states.

By appealing to our common sense, we could declare for example, that a state like

> $HRS\overline{W}$ standing for "it is hot, it is raining and snowing, and it is not wet"

is impossible, while a state like

> $H\overline{RSW}$ standing for "it is hot, it is neither raining nor snowing, and it is not wet"

would be very possible indeed.

Since we don't talk about states but valuations in logical terminology, we also say

[2]For the traditional logic bit variables have the status of (elementary) assertoric sentences or propositions. (Accordingly Wittgensteins *facts* are true assertoric sentences.) But we would like to motivate the reader, to paraphrase bit variables not only by assertoric sentences ("it is hot", "it is raining" and so on), but also by substantives ("heat", "rain" and so on) or verbs ("to heat", "to rain" and so on).

— ZERO VALUATION if a state is impossible and
— UNIT VALUATION for a possible state in a world.

Besides we correctly talk about a (ASSERTORIC LOGICAL) THEORY instead of a world.

So a theory is completely defined by
— a set of n bit variables,
— the set of all 2^n valuations given by the bit variables, and
— a separation of this set of valuations into
  — a set of zero valuations and
  — a set of unit valuations.

Accordingly our example theory is defined by:
— the 4 bit variables H, R, S, W
— the resulting $2^4 = 16$ different valuations

$$\begin{array}{llll}
\overline{H}\overline{R}\overline{S}\overline{W} & H\overline{R}\overline{S}\overline{W} & \overline{H}R\overline{S}\overline{W} & HR\overline{S}\overline{W} \\
\overline{H}\overline{R}S\overline{W} & H\overline{R}S\overline{W} & \overline{H}RS\overline{W} & HRS\overline{W} \\
\overline{H}\overline{R}\overline{S}W & H\overline{R}\overline{S}W & \overline{H}R\overline{S}W & HR\overline{S}W \\
\overline{H}\overline{R}SW & H\overline{R}SW & \overline{H}RSW & HRSW
\end{array}$$

— and of these valuations
  — 12 are zero valuations

$$\begin{array}{llll}
\overline{H}R\overline{S}\overline{W} & HR\overline{S}\overline{W} & \overline{H}\overline{R}S\overline{W} & H\overline{R}S\overline{W} \\
\overline{H}RS\overline{W} & HRS\overline{W} & \overline{H}\overline{R}\overline{S}W & H\overline{R}\overline{S}W \\
\overline{H}R\overline{S}W & H\overline{R}SW & \overline{H}RSW & HRSW
\end{array}$$

  — and the remaining 4 are unit valuations

$$\begin{array}{llll}
\overline{H}\overline{R}\overline{S}\overline{W} & H\overline{R}\overline{S}\overline{W} & HR\overline{S}W & \overline{H}\overline{R}SW
\end{array}$$

If we use the following abbreviations:
— Θ for a set of bit variables,
— $Val\Theta$ for the set of valuations generated by this set of bit variables,
— Ω_0 for the set of all zero valuations, and
— Ω_1 for the set of all unit valuations,

then a theory has the following form:

$(\Theta, Val\Theta, \Omega_0, \Omega_1)$

But this notation would be much too awkward.

First we can either omit Ω_0 or Ω_1, because if one of the sets is given, the other is completely determined. Thus instead of $(\Theta, Val\Theta, \Omega_0, \Omega_1)$ we could write $(\Theta, Val\Theta, \Omega_0)$ or $(\Theta, Val\Theta, \Omega_1)$.

Besides the set $Val\Theta$ is completely determined by the given Θ, so we can further abbreviate the notation for a theory to (Θ, Ω_0) or (Θ, Ω_1).

But even this way of writing is very impractical in general, because, different to our example, the sets Ω_0 and Ω_1 usually consist of very many very long valuations, which to specify explicitly would be too laborious and complex. We will use a common and more efficient notation to separate the set $Val\Theta$ into the disjunct subsets Ω_0 and Ω_1, that is a formula θ. With such a formula a theory is formally defined by the expression (Θ, θ). This is the common notation and we will use it in a modified version and write

$[\theta, \Theta]$

instead of (Θ, θ) for a theory.

And if the set of all bit variables occuring in θ is exactly the set Θ, we will even allow the omission of Θ and simply write

$[\theta]$

for the same theory.

Besides it can be easily shown that for every expression $[\theta, \Theta]$ of a theory with a finite set Θ of bit variables, a formula θ' exists such that $[\theta, \Theta] = [\theta]$. So it does not matter which of the two notations we actually use.

But first of all we need to define what we mean by a formula.

1.4 Formulas

For a set Θ of bit variables we define the set $Form\Theta$ of all FORMULAS generated by Θ as follows:

— Each of the bit variables and each bit value is a formula, and

— if already given formulas are combined to new expressions by using the JUNCTORS

$\neg$ standing for "not ...",

$\wedge$ standing for "... and ...",

$\vee$ standing for "... or ...",

$\rightarrow$ standing for "if ..., then ...", and

$\leftrightarrow$ standing for "... if and only if ..."

then the result is a formula as well.

Examples for formulas generated by $\{H, R, S, W\}$ are:

— R standing for "it is raining".

— 0 standing for "that is impossible".

— 1 standing for "this is always the case".

— $R \rightarrow 0$ standing for "if it rains, then it is impossible". In other words: "it does not (never) rain".

— $\neg(R \wedge S)$ standing for "it cannot rain and snow (at the same time)".

— $W \leftrightarrow (R \vee S)$ standing for "it is wet if and only if it rains or snows".

— $R \rightarrow H$ standing for "if it rains, then it is hot".

— $S \rightarrow \neg H$ standing for "if it snows, then it is not hot".

The process of generating new formulas from already given ones can be continued on and on. The last four formulas can be combined by using the junctor "... and ..." to[3]

$$\neg(R \wedge S) \wedge (W \leftrightarrow (R \vee S)) \wedge (R \rightarrow H) \wedge (S \rightarrow \neg H)$$

So there are infinitely many formulas that can be generated from any given set of bit variables.

[3] Some superfluous parantheses are left away, the common conventions of parenthesing and abbreviating shall hold.

We stated that a formula θ of $Form\Theta$ would separate the set $Val\Theta$ of all valuations into the sets of zero and unit valuations. For instance

— $R \wedge S$ demands that in each unit valuation R as well as S have to occur. (And so neither $\overline{R}$ nor $\overline{S}$ must occur.) Thus the theory $[R \wedge S, \{H, R, S, W\}]$ has exactly the four following unit valuations:

$$\overline{H}RS\overline{W} \quad HRS\overline{W} \quad \overline{H}RSW \quad HRSW$$

— $\neg(R \wedge S)$ on the other hand would demand, that in no unit valuation both R and S may occur. So the theory $[\neg(R \wedge S), \{H, R, S, W\}]$ would have exactly the four unit valuations different from the four just mentioned ones.

— $S \rightarrow \neg H$ states for every unit valuation, that if S occurs in it, then $\overline{H}$ has to occur in it as well. In other words: zero valuations are exactly those valuations containing S as well as H.

— $W \leftrightarrow (R \vee S)$ demands of every unit valuation, that it contains W if and only if R or S occur in it as well. In this way the theory $[W \leftrightarrow (R \vee S), \{H, R, S, W\}]$ has precisely the following eight unit valuations:

$$\overline{HRSW} \quad H\overline{RSW} \quad \overline{H}R\overline{S}W \quad HR\overline{S}W$$
$$\overline{HR}SW \quad H\overline{R}SW \quad \overline{H}RSW \quad HRSW$$

Our standard example of a theory with the set $\{H, R, S, W\}$ of bit variables and the four unit valuations $\overline{HRSW}$, $H\overline{RSW}$, $HR\overline{S}W$, $\overline{HR}SW$ can be formalized as

$$[\theta_w, \Theta_w]$$

where

$$\theta_w = \neg(R \wedge S) \wedge (W \leftrightarrow (R \vee S)) \wedge (R \rightarrow H) \wedge (S \rightarrow \neg H)$$
$$\Theta_w = \{H, R, S, W\}$$

(The index "w" stands for "weather".)

Since Θ_w is exactly the set of bit variables occuring in θ_w, the theory $[\theta_w, \Theta_w]$ can also be written as $[\theta_w]$.

Given a finite set of n bit variables.

— The number of valuations generated by this set is finite. (Namely there are 2^n different ones.)

— The set of all formulas generated by this set is infinite, but

— the set of all theories having this set as their bit variable set is finite again. (Namely there are 2^{2^n} different ones.)

This is, because many formulas exist that describe the same theory each. For instance

$$\begin{aligned} &[\neg(R \wedge S)] \\ &= [\neg R \vee \neg S] \\ &= [(\neg R \wedge \neg S) \vee (R \wedge \neg S) \vee (\neg R \wedge S)] \\ &= [R \rightarrow \neg S] \end{aligned}$$

In fact for every theory infinite many formulas exist to describe it. Each two of these formulas are said to be EQUIVALENT.[4]

[4] Frege expressed the phenomenon of the equivalent formulas in the following way:

1.5 Assertions

Now we try to develop a concept of an assertion. We will take the principle, commonly named after Aristotle, as a guide in saying:

An assertion is an expression, that is either true or false.

We call TRUE and FALSE the TRUTH VALUES.

Thus to define assertions we have to

— first define, what kind of expression is meant, and
— second define, in what way a truth value is assigned to this expression.

1.5.1 The definition of the assertion as a formula

Frequently the concept of an assertion is set equal to a formula. Hence each formula has to be either true or false. But what decides, if for example the formula $\neg(R \wedge S)$ is true or false?

If we represent the formula $\neg(R \wedge S)$ as the theory $[\neg(R \wedge S)]$, this theory has zero valuations (namely RS) as well as unit valuations (namely $\overline{R}\overline{S}$, $R\overline{S}$,$\overline{R}S$). But to say that the assertion $\neg(R \wedge S)$ would be partly false and partly true would violate the Aristotelian principle, since this demands a clear decision.

But the attempt to define assertions as formulas could be rescued if we say:

— a formula φ shall be true, if φ is a TAUTOLOGY, in other words:
— if all valuations of the theory $[\varphi]$ are unit valuations.

Consequently $\neg(R \wedge S)$ is a false formula, i. e. a false assertion, while tautologic formulas like $R \vee \neg R$ and $R \rightarrow (R \vee S)$ are true assertions.

So the concept of assertion can be identified by formulas, but this definition would be quite impractical. We will make another proposal.

1.5.2 The definition of the assertion as a pair of theories

Instead of assigning an absolute truth value to a fomula, we are going to relativate the truth concept by saying:

A formula is true or false according to a theory.

The formula $\neg(R \wedge S)$ for instance would be true according to our standard theory $[\theta_w]$, because in this world a state where it is raining and snowing at the same time, is impossible and thus $\neg(R \wedge S)$ is true in every case. On the other hand there are theories according to which the formula $\neg(R \wedge S)$ would

Indeed the formulas are different as written formulas, but their SENSE (SINN)*, their* THOUGHT (GEDANKE) *is equal.*

(In fact he used the term SENTENCE (SATZ) instead of formula.)

What Frege called the sense of a formula, we will call the PROPOSITION. We write $\langle\theta\rangle$ for the proposition of a formula θ and define it to be the equivalence class of all formulas equivalent to θ.

An essential point of the (assertion) logic developed here, is to elaborate the difference between a proposition $\langle\theta\rangle$ and a theory $[\theta]$. A difference Frege (due to his platonic, transcendental concept of logic) was not able to notice.

be wrong. (For example, if S is not associated with "it is snowing", but with "the sun is shining" and the standard theory would be modified in the obvious way.)

Thus an assertion would be a pair $([\theta], \varphi)$ of a theory $[\theta]$ and a formula φ.

But our concern shall not be the formula φ itself, but only what it expresses. That means, we will define two expressions $([\theta], \varphi_1)$ and $([\theta], \varphi_2)$ to be the same assertion, if φ_1 and φ_2 are equivalent. We achieve this by defining an assertion not as a pair $([\theta], \varphi)$, but as a pair $([\theta], [\varphi])$. In this way two assertions $([\theta], [\varphi_1])$ and $([\theta], [\varphi_2])$ are equal if, and only if φ_1 and φ_2 are equivalent (and have the same bit variables).[5]

Besides we need to make the restriction that $[\varphi]$ may only contain bit variables which occur in $[\theta]$.[6]

The final definition sounds as follows:

> An ASSERTION is a pair of theories, where the second theory contains only bit variables, which occur in the first one as well.

Abbreviated we also write

— $[\theta \mid \varphi]$ instead of $([\theta], [\varphi])$,

— $[\theta, \Theta \mid \varphi, \Phi]$ instead of $([\theta, \Theta], [\varphi, \Phi])$

and so on for assertions.

1.5.3 The truth value of an assertion

Now we still miss a criterion to decide whether an assertion is true or false.

An assertion $[\theta \mid \varphi]$ is TRUE if and only if all unit valuations of $[\theta]$ are in accordance to what is expressed by φ. Otherwise the assertion is said to be FALSE.

To demonstrate this we take our standard theory again

$$[\theta_w] = [\neg(R \wedge S) \wedge (W \leftrightarrow (R \vee S)) \wedge (R \rightarrow H) \wedge (S \rightarrow \neg H)].$$

The four unit valuations of $[\theta]$ are

$$\overline{H}\,\overline{R}\,\overline{S}\,\overline{W} \quad H\overline{R}\,\overline{S}\,\overline{W} \quad HR\overline{S}W \quad \overline{H}\,\overline{R}SW$$

With this we discuss the following examples of assertions:

— Let φ_0 be the formula $R \vee \neg R$, standing for "it is raining or it is not raining". In each of the four unit valuations either R or $\overline{R}$ occurs. Consequently "it is raining or it is not raining" is true according to our standard theory: $[\theta_w \mid \varphi_0]$ is a true assertion.

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[5] According to this reasoning it would have been more appropriate and sufficient to define an assertion not as a pair $([\theta], [\varphi])$ of theories, but as a pair $([\theta], \langle\varphi\rangle)$ of a theory and a proposition. This is correct, especially since this difference is irrelevant in connection with the concept of truth. But when it comes to the concept of meaning this difference becomes crucial. This will become clear in the discussion about the so–called opposition principle later on.

[6] Otherwise $[\varphi]$ would "talk about things (bit variables)" not known in $[\theta]$ and a decision about the truth of the assertion is undetermined. (At least in the general case that one of the unknown bit variables is VALENT in $[\varphi]$.) If this restriction is repealed, we talk about a MESSAGE. Then an assertion is a special message.

- Let φ_1 be the formula $\neg(R \wedge S)$, standing for "it cannot rain and snow (at the same time)". None of the unit valuations contains R and S at the same time. Thus $\neg(R \wedge S)$ is the case for all unit valuations: $[\theta_w \mid \varphi_1]$ is a true assertion.
- Let φ_2 be the formula R, standing for "it rains". Now there is one unit valuation of $[\theta_w]$, namely $HR\overline{S}W$, in accordance to R, but there are also the three unit valuations $\overline{HRSW}$, $H\overline{RSW}$, and $\overline{HR}SW$ for which R is not the case, but $\overline{R}$. Hence $[\theta_w \mid R]$ is a false assertion.
- Let φ_3 be the formula $R \rightarrow H$, standing for "if it rains, then it is hot". That says: if R occurs in a unit valuation, H must occur in it as well. (And if $\overline{R}$ occurs, it does not matter if H or $\overline{H}$ occurs.) This is for all four unit valuations the case: $[\theta_w \mid \varphi_3]$ is a true assertion.

In example φ_2 we saw that "it rains" is false according to our theory, although rain is a possible state. So an assertion $[\theta \mid \varphi]$ does not say

"φ is possibel in the theory $[\theta]$"

but rather says

"φ is always the case in the theory $[\theta]$".

The common terminology for that is

φ is VALID IN $[\theta]$

or

φ DERIVES FROM $[\theta]$

and by using the derivation symbol $\Rightarrow$, we also write

$[\theta] \Rightarrow \varphi$

or

$\theta \Rightarrow \varphi$

so that we finally have:

An assertion $[\theta \mid \varphi]$ is true if, and only if $\theta \Rightarrow \varphi$.

1.5.4 The truth function

Whether a given assertion is true or false can efficiently and reliable be decided by a computer. A great number of methods exist to solve this problem. We will express this here by a functional notation, without going deeper into one of this methods.

The TRUTH FUNCTION that assigns a truth value

$$truth[\theta \mid \varphi] = \begin{cases} 1 & \text{if } [\theta \mid \varphi] \text{ is true} \\ 0 & \text{if } [\theta \mid \varphi] \text{ is false} \end{cases}$$

to an assertion $[\theta \mid \varphi]$ can be calculated by a computer.

Chapter 2

The development of a concept of meaning

2.1 The basic principle

For the traditional semantic meaning is a relation between two distinct ontological spheres.[1] Typical examples are:

- — The relation between *language* and *world*:
  - — The substantive "snow" means this white cold matter.
  - — The verb "snow" means the downfall of snow–flakes.
  - — The sentence "it is snowing" means the fact that it is snowing.
- — The relation between *language* and *thought*,
- — between *spoken* and *written language*,
- — between *one* and *another one (natural) language*,
- — between *notes* and *music*.

The basic idea now for the development of an alternative logical concept of meaning is, to doubt the existence of a "world" beyond the "language". There is only a system of signs. And meaning is a relation between two signs, both of the same system.

In terms of the formally introduced concepts of the assertion logic given here so far, a system of signs is constituted by a theory $[\theta]$ and the signs are all assertions of the form $[\theta \mid \varphi]$.

So we formulate the idea of an alternative semantic in the following FUNDAMENTAL PRINCIPLE:

> For a given theory $[\theta]$ the meaning of an assertion $[\theta \mid \varphi]$ is an assertion $[\theta \mid \mu]$.

[1] By more or less formalizing these pair of categories each as a so–called OBJECT and META LANGUAGE, the idea and possibility of modern semantics as a mathematic–logical science is grounded. (Manifested in particular by A. Tarski *Über den Wahrheitsbegriff in den formalisierten Sprachen*, 1935.)

2.2 The meaning function

The fundamental principle is included in the functional definition of the meaning concept via the so–called meaning function.

A MEANING FUNCTION determines for every assertion $[\theta \mid \varphi]$ a theory $[\mu]$.

We write this in the form

$meaning[\theta \mid \varphi] = [\mu]$

and say

— $[\mu]$ is the MEANING OF $[\theta \mid \varphi]$, or

— $[\mu]$ is the MEANING OF $[\varphi]$ IN $[\theta]$.

In fact the basic principle did not define a theory $[\mu]$, but the assertion $[\theta \mid \mu]$ to be the meaning of $[\theta \mid \varphi]$. But since $[\theta]$ does not change in the result, it is left away for the sake of brevity.

Now there is still to be defined, how a meaning function computes a theory $[\mu]$ for a given assertion $[\theta \mid \varphi]$.[2]

2.3 The non–discriminating principle

The meaning function already implies the NON–DISCRIMINATING PRINCIPLE of our alternative meaning concept:

Every assertion has a meaning.

This shall only be emphasized explicitly because some kind of logics are forced by their own concept to specify a criterion to be able to exclude senseless or paradox expressions apriori, which are forbidden in the calculus.

2.4 The lexicon

Taking the perspective of the traditional semantic the fundamental principle seems awkward. But in everyday life there are phenomena, where the meaning concept is used according to this principle. A typical example is the monolingual dictionary or lexicon.

A lexicon is, one could say, a list of definitions of possibly all the vocabulary or basic concepts which are available to describe a world.

A current English lexicon for instance is a list of definitions of possibly all English words, which are available for the English speaking to describe their common, linguistically constituted world.

Such a definition is a pair (D, δ) of

[2]Of course there are again infinitely many different formulas μ, μ_1, μ_2, ... that would represent the one theory $[\mu] = [\mu_1] = [\mu_2] = ...$ and a meaning function has to select one to be well–defined. There are a lot of ways to choose a unique, so–called CANONIC NORMAL FORM from infinite many equivalent formulas, for instance the CANONIC DISJUNCTIVE NORMAL FORM. But we will occasionally prefer a more intuitive or shorter form.

— first, a defined word D,
— second, a defining concept or sentence δ.

To make sure that the definition is not circular, it is demanded that the word D must not occur in it's explanation δ.

The definition is read as
— ""D" is defined by δ" or
— ""D" means δ".

A lexicon for our little world of the standard example could contain the following definitions:
— "Rain" means wet, hot weather.
— "Snow" means wet and not hot weather.
— "Wet weather" means rain or snow.

2.5 Meaning as defining

We will make an attempt to use the idea of the definition for a precise determination of our meaning concept. Therefore we need to formalize the definition concept in terms of the assertion logic and make the following statement:

A DEFINITION in a theory $[\theta]$ is a formula $D \leftrightarrow \delta$ with
— $D \leftrightarrow \delta$ is valid in $[\theta]$.
— D is a bit variable form $[\theta]$ and δ contains only bit variables form $[\theta]$ which are different from D.

If we formalize the mentioned examples in this way, we actually get three definitions, namely
— $R \leftrightarrow W \wedge H$
— $S \leftrightarrow W \wedge \neg H$
— $W \leftrightarrow R \vee S$

Until now a definition is restricted to a bit variable D. But we need a formal definition concept for all formulas φ, that can be generated from the bit variables of $[\theta]$. Thus we extend the definition of the definition in the obvious way:

A DEFINITION in a theory $[\theta]$ is a formula $\varphi \leftrightarrow \mu$ with
— $\varphi \leftrightarrow \mu$ is valid in $[\theta]$.
— φ and μ contain only bit variables from $[\theta]$, but no common ones.

If $\varphi \leftrightarrow \mu$ is a definition in a theory $[\theta]$ we also say:
— μ DEFINES φ IN $[\theta]$, or
— $[\mu]$ DEFINES $[\varphi]$ IN $[\theta]$, or
— $[\mu]$ DEFINES $[\theta \mid \varphi]$.

So we make the following attempt to develop a meaning concept out of the definition concept:

The meaning of an assertion $[\theta \mid \varphi]$ is a theory $[\mu]$ with:
— $\varphi \leftrightarrow \mu$ is valid in $[\theta]$ and
— φ and μ contain only bit variables from $[\theta]$, but no common ones.

For the mentioned examples we get:

- $[W \wedge H]$ is the meaning of $[R]$ in $[\theta_w]$.
- $[W \wedge \neg H]$ is the meaning of $[S]$ in $[\theta_w]$.
- $[R \vee S]$ is the meaning of $[W]$ in $[\theta_w]$.

But in fact this first attempt to develop a concept of meaning according to the fundamental principle must fail. Because a closer look shows:

- First, not every $[\varphi]$ is defineable in $[\theta]$.
  For example, there is no definition for H in $[\theta_w]$, no μ, such that $H \leftrightarrow \mu$ is a definition.
- Second, there are theories $[\varphi]$ in $[\theta]$, which are defineable in multiple different ways.
  For example $[W \wedge H]$, $[W \wedge H \wedge \neg S]$, and $[W \wedge \neg S]$ are different theories, each defining $[R]$ in $[\theta_w]$.

Consequently the meaning function cannot be defined like that.

2.6 Meaning as verifying

We have seen how the principle of definition can be a model for a concept of meaning according to the fundamental principle. But a solution, a well–defined concept of meaning was not possible that way. We will now present another attempt, also in accordance with the fundamental principle: the verification. That tries to formalize an idea Peirce[3] describes as:

> *The meaning of a statement is the method of empirically confirming or infirming it.*

Wittgenstein expresses something similar in *Tractatus logico–philosophicus*, statement 4.024:

> *Understanding a sentence, means, to know what the case is, if it is true.*

In a first approach we formulate this idea for our meaning concept in terms of the assertion logic as follows:

> The meaning $[\mu]$ of an assertion $[\theta \mid \varphi]$ indicates a case such that $[\theta \mid \varphi]$ is true.

And we will say

- $[\mu]$ VERIFIES $[\theta \mid \varphi]$ or
- $[\mu]$ VERIFIES $[\varphi]$ IN $[\theta]$,

if $[\mu]$ is a case such that $[\varphi]$ is true in $[\theta]$.

Again we consult our standard world:

- In what case is it wet?
  - First solution: In case it rains.
  - Second solution: In case it snows.
  - Third solution: In case it rains or snows.
  - Fourth solution: In case it is warm and it rains and it does not snow.

  That means that each of the four theories $[R]$, $[S]$, $[R \vee S]$, $[H \wedge R \wedge \neg S]$ verifies the theory $[W]$ in $[\theta_w]$.

[3]Quoted from Quine *Two dogmas of empriricism*, 1951.

So there are quite a few possibilities (even more than four) to verify $[\theta_w \mid W]$ although $[\theta_w \mid W]$ itself is false.

— Theories that verify the false assertion $[\theta_w \mid R]$ are for instance: $[\neg S \wedge W]$,$[H \wedge W]$, $[H \wedge \neg S \wedge W]$.

— Theories that verify $[\theta_w \mid S \vee W]$ are for instance: $[R]$, $[H \wedge R]$.

Anyway, we still miss a formal concept of the idea, that the meaning, that is the verifying theory, "indicates a case such that the assertion is true". We want to achieve that $[\varphi]$ becomes true in $[\theta]$ by giving a state $[\mu]$ for which that is the case. That means, the truth basis for $[\varphi]$ is not any longer $[\theta]$ alone, but the theory $[\theta]$ *and* the theory $[\mu]$; that is the theory $[\theta \wedge \mu]$.

For assertions $[\theta \mid \varphi]$ and $[\theta \mid \mu]$ we say:[4]

$[\mu]$ verifies $[\varphi]$ in $[\theta]$, if $[\theta \wedge \mu \mid \varphi]$[5] is true.

In fact all the just mentioned examples submit to this formal definition.

Different to the definition concept, for the verification holds:

For every assertion $[\theta \mid \varphi]$ there is a $[\mu]$ that verifies $[\varphi]$ in $[\theta]$.

Similar to the definition concept, for verification holds too:

For every assertion $[\theta \mid \varphi]$ there are in general several different theories that verify $[\varphi]$ in $[\theta]$.

But we can remove this ambiguity by selecting a distinguished one out of the set of all verifying theories. For example the one we call MAXIMAL and which always exists in a unique way.

But first we will turn our attention to another phenomenon.

2.7 The opposition principle

The presentation of our semantic started from the so–called fundamental principle. But next to it is a second principle, as central and radical different from the established semantic, that guided the search for a new, alternative concept. We want to call this idea the OPPOSITION PRINCIPLE.

The term OPPOSITION is drawn from linguistic structuralism, especially from the structuralistic phonology.[6]

This stream has stimulated the idea that the semantic of a sign is not a pretended something to which this sign would point to, but rather a demarcation against all the other signs of the system of signs different from this chosen sign. In other words, meaning is no longer thought of as an act of identification, but one of differencing.

To illustrate such kind of differential semantic we imagine a system of signs consisting only of the signs "red", "yellow", and "blue". In this simple colour

[4] In fact this is not the final definition of the verification concept. But all statements further made here will still hold.

[5] Another equivalent formulation would be $[\theta \mid \mu \rightarrow \varphi]$ instead of $[\theta \wedge \mu \mid \varphi]$.

[6] In a mathematical terminology the opposition principle would have rather been called COMPLEMENTARY PRINCIPLE.

language the expression "this is red" *means* "this is neither yellow nor blue".

Now we are going to formulate the opposition principle in terms of our logic developed so far.

Let $[\theta \mid \varphi]$ be a given assertion and

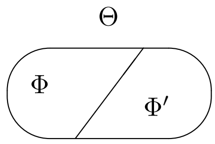
— Θ the set of bit variables in θ and

— Φ the set of bit variables in φ

so that

$$[\theta \mid \varphi] = [\theta, \Theta \mid \varphi, \Phi].$$

The defintion of the assertion concept determines that Φ is a subset of Θ as the following picture shows:



We call Φ' the set of all the bit variables of Θ not occuring in Φ.

Indirectly the assertion $[\theta \mid \varphi]$ is concerned with all the bit variables of Θ, but directly it only makes a statement about the bit variables of Φ. If we think of meaning as a verification, so that $[\mu]$ designates the circumstances under which $[\theta \mid \varphi]$ becomes true, it is reasonable to demand that $[\mu]$ is restricted to the bit variables of Φ'. Because everything concerning the bit variables form Φ is already determined by $[\varphi]$.

So we formulate the OPPOSITION PRINCIPLE for our meaning concept as follows:

> The meaning of an assertion $[\theta, \Theta \mid \varphi, \Phi]$ is a theory of the form $[\mu, \Phi']$, where Φ' is the set of all bit variables of Θ that don't occur in Φ.[7]

Let us take a closer look to the example

$$[\theta_w \mid R]$$

where $[\theta_w]$ designates our standard example again. So the set of bit variables in $[\theta_w]$ is again

$$\Theta_w = \{H, R, S, W\}$$

the set of bit variables in $[R]$ is $\{R\}$ and thus

$$[\theta_w \mid R] = [\theta_w, \{H, R, S, W\} \mid R, \{R\}].$$

The opposition requires of a potential meaning of $[\theta_w \mid R]$ that this must have the form

$$[\mu, \Phi']$$

where Φ' are the bit variables of Θ_w, not occuring in $\{R\}$, thus

$$\Phi' = \{H, S, W\}$$

and the formula μ may have no other bit variables than H, S, and W. For instance

$$[W \wedge H \wedge \neg S] = [W \wedge H \wedge \neg S, \{H, S, W\}]$$

[7] At this point it becomes clear why we used the theory concept to define assertions, and not propositions. The opposition cannot be formulated in a pure propositional logic.

would be a theory that satisfies the opposition principle. Besides it defines and verifies the assertion. This is also done by

$[W \wedge H] = [W \wedge H, \{H, W\}]$

except that it violates the opposition principle. But if this theory is modified to

$[W \wedge H, \{H, S, W\}]$

it would satisfy the opposition principle and defines and verifies the assertion.[8]

By the way, in fact we already met a special variant of the opposition principle earlier on for the definition, namely the condition that a definition must not be circular. There it was demanded that μ and φ must not contain common bit variables, if μ ought to define φ in $[\theta]$.

So it is well motivated to postulate the opposition principle for the meaning concept, both for designing it as a definition and as a verification.

2.8 The true meaning function

If we now combine the presented ideas we finally reach a fully determined meaning concept.

First we modify the verification concept by including the opposition principle:

A theory $[\mu]$ VERIFIES an assertion $[\theta \mid \varphi]$ if and only if

— $[\theta \wedge \mu \mid \varphi]$ is true and

— $[\mu]$ contains exactly the bit variables of $[\theta]$, that don't occur in $[\varphi]$.

It can be shown that for every assertion $[\theta \mid \varphi]$ there is exactly one theory $[\mu_{true}]$, which ABSOLUTE MAXIMAL verifies it, that means:

> All unit valuations of any other verifying theory $[\mu]$ are included in $[\mu_{true}]$ as well. (In other words: $\mu \Rightarrow \mu_{true}$.)

Because every assertion has exactly one such absolute maximal verifying theory, we can define a meaning function by using this as it's mapping. We will call it the TRUE MEANING FUNCTION due to a remarkable property we are going to discuss in the following final paragraph.[9]

The true meaning function assigns a theory

$truemean[\theta \mid \varphi] = [\mu_{true}]$

to every assertion $[\theta \mid \varphi]$ such that the assertion is absolute maximal verified.

[8] In a theory like $[W \wedge H, \{H, S, W\}]$ the bit variables H and W are called VALENT, S on the other hand is INVALENT. The need to declare invalent bit variables just to fulfil the opposition principle, seems unnecessary complicated at first sight. But we shall not discuss this point any further here.

[9] It shall be mentioned that the true meaning function is not the only reasonable meaning function. For instance, there is another one we call the ABSOLUTE SATISFYING MEANING FUNCTION, for which, next to the here developed principles (opposition, verification, and definition) also the so–called PRINCIPLE OF ABSOLUTE SATISFIABILITY holds. It's results, the ABSOLUTE SATISFIABLE MEANINGS, often better match the naive understanding than the true meanings. The relation of these two meaning function resembles the relation of the mathematical and the naive idea of the subjunction $\rightarrow$. But the disadvantage of the absolute satisfying against the true meaning function is, that it does not fulfil the powerful PRINCIPLE OF EMBEDDING THE THUTH INTO THE MEANING CONCEPT (see below) any more.

We say
— $[\mu_{true}]$ is the TRUE MEANING OF $[\theta \mid \varphi]$ or
— $[\mu_{true}]$ is the TRUE MEANING OF $[\varphi]$ IN $[\theta]$.

How the true meaning is actually inferred form a given assertion shall not further be discussed here. Instead we will give some examples in relation to our standard example $[\theta_w]$:
— $[(R \wedge \neg W) \vee (\neg R \wedge W) \vee (\neg H \wedge R)]$ is the true meaning of $[S]$.
— $[R \vee S, \{H, R, S\}]$ is the true meaning of $[W]$, and the other way round:
— $[W]$ is the true meaning of $[R \vee S, \{H, R, S\}]$.
— $[W, \{H, W\}]$ is the true meaning of $[R \vee S]$.
— $[H \wedge \neg H \wedge W \wedge \neg W] = [0, \{H, W\}]$ is the true meaning of $[R \wedge S]$.
— $[H \vee \neg H \vee W \vee \neg W] = [1, \{H, W\}]$ is the true meaning of $[\neg(R \wedge S)]$.
— $[H \vee \neg H] = [1, \{H\}]$ is the true meaning of $[R \vee S \vee \neg W]$.
— $[0]$ is the true meaning of $[H \wedge R \wedge S \wedge W]$.

When we first made an attempt to develop a meaning concept via the definition concept, we had to realize that the intended way was impossible since not every assertion possesses a defining theory. But the true meaning function has the following property; it satisfies the so–called DEFINITION PRINCIPLE:

If an assertion has a defining theory at all, the true meaning is such one.

The reader may try to illustrate this for the just mentioned examples, if and when $\theta_w \Rightarrow \varphi \leftrightarrow \mu_{true}$ holds.

2.9 Embedding the truth into the meaning concept

In general traditional semantic builds the truth on the meaning concept: only after the meaning of an assertion is determined, it can be decided if it is true or false.

In the construction of our meaning concept this epistemological order turns the other way round: first a concept of an assertion and it's truth value had to be present to derive the meaning from it; the meaning function needed the truth function (the derivation concept); meaning presupposed truth.

So it is even more remarkable that on the other hand the truth concept is, so to say *embedded*[10] into our concept of the true meaning, namely in the following way, the so–called PRINCIPLE OF EMBEDDING THE TRUTH INTO THE MEANING CONCEPT:[11]

[10]Due to the richer historic and the semantic connotations of the German term AUFHEBUNG against EMBEDDING, each highlighting a certain interesting aspect of the relation between truth and meaning, we would have preferred to give the following principle this German name, if it would not have let to ungrammatical sounding constructions handling it in English.

[11]What is in logic meanwhile generally called the TRUTH VALUE OF A FORMULA (here modified: OF AN ASSERTION), was named the MEANING (BEDEUTUNG) OF A SENTENCE in Freges semantic. (Although there is hardly a second such well composed and consequent semantic — taken out of its context the terminology sounds awkward and this might be the reason it didn't survive. In fact the term MEANING is no term of modern formal logic at all.) So the

Let

— $[\theta \mid \varphi]$ be an assertion,

— $\tau := truth[\theta \mid \varphi]$ the truth value and

— $[\mu] := truemean[\theta \mid \varphi]$ the true meaning of it,

then it holds that:

μ and 1 are equivalent if and only if τ is 1.

In short:

The true meaning of an assertion is a tautological theory if and only if the assertion is true.

This phenomenon motivated the title TRUE MEANING (FUNCTION).

The principle of embedding is confirmed for the examples given above:

— $[1 \mid \{H, W\}]$ is the true meaning of $[\neg(R \wedge S)]$ in $[\theta_w]$, since $[\neg(R \wedge S)]$ is true in $[\theta_w]$.

— $[1 \mid \{H\}]$ is the true meaning of $[R \vee S \vee \neg W]$ in $[\theta_w]$, because $[R \vee S \vee \neg W]$ is true in $[\theta_w]$.

The six remaining example assertions are not true and their true meanings are no tautological theories.

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principle of embedding might have adequate been called the EMBEDDING OF FREGES MEANING CONCEPT.